# Improved Magnitude Weighting Technique for Wireless Sensor Network Beamformer

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## Abstract

The wireless sensor node consumes maximum energy for data communication which calls for highly directive beams for point to point communication applications to reduce its energy consumption in a Wireless Sensor Network (WSN). This paper presents the synthesis of a wideband beamformer, which is highly directive with reduced sidelobe level (SLL). It supports wide-angle beam steering which makes it suitable for Uniform Linear Array (ULA) WSN. An expression is obtained for weighting the array factor (AF) of the wideband beamformer to achieve beampattern model with desired SLL and Half Power Beam-Width (HPBW) specifications. Compared to Taylor and DPSS techniques, the proposed beamformer shows higher directivity for a given number of elements. It has lower and tapered sidelobes, hence better First Null to Last Null (FNLN) ratio. The proposed beamformer shows improved performance in wideband of interest i.e. 1-3GHz and basically, it is a non-optimization approach.

## Introduction

Wireless Sensor Networks (WSN) are the systems that comprise a large number of random sensor nodes that can observe, collect and process the desired information and transmit only the required data. They are an energy-constrained system. The power-constrained sensor node involves long distance data transmission. This issue can be resolved considerably by using beamforming techniques. WSN based on array beamforming, wherein a narrow main beam is directed towards the specified user direction, while the nulls and low sidelobes are directed towards the interferers. Different window functions like Dolph–Chebyshev, Hamming, Kaiser, etc. can be used to taper the amplitudes of the weights, of the tapered beamformer employed in WSNs [1]. Beamforming has been introduced to WSN to increase the transmission range. Nodes in the WSN have omnidirectional antennas and to prevent the wastage of power in all directions, the nodes work as a cluster and act collaboratively. Malik et al. [2] present a novel method to minimize the beamforming performance errors, by determining the optimum sensor nodes coordination. Dolph [3] has shown that to achieve the low SLLs some tradeoff has to be done between beam-width and overall gain, known as minimax response and employs Chebyshev polynomials to yield the optimum array pattern for the ULAs. In WSNs, the power of node is drawn by its three activities namely, data processing, communication and sensing. Among these operations, the node utilizes maximum energy in data communication. The proper antenna design therefore plays a vital part in minimizing the node's energy consumption. Directional antennas radiate towards a specific direction, thus they reduce the energy consumption and improve the energy efficiency of the system. Therefore directional antennas can be used in WSN static end nodes. Michalopoulou et al. [4] investigated the reliability of WSN systems with directional antennas and their impact on energy efficiency. They have compared the performance of directional antennas in outdoor environment with the omnidirectional antennas at 2.45 GHz, and it showed better node performance with regards to energy efficiency and node lifetime. To form the angular diversity in WSNs, Yang et al. [5] employed directional antennas on motes. It can manage with uncertainties in the channels with improved  energy conservation and bandwidth utilization and also increases the mean received signal strength. Therefore, the infrastructure of the WSN can be enhanced with directional antennas and a switching network. Jiang et al. [6] showed the minimum energy broadcast problem in WSN, which is subjected to the constraints on directional antennas and available energy, by solving the optimization problem using a proposed algorithm. A combination of directional antennas and multi-channel were used to save radiating energy and reduce interference in wireless communication. Practical directional antennas are categorized into two, namely sectored antenna and smart antenna. The sectored antenna has fixed sectors and beam- width, while smart antenna can steer its direction to any desired direction, and also its beam-width can be adjusted from a minimum angle to 360 [0]. Either smart antennas or sectored antennas are used in finding the solutions for energy-efficient broadcasting. Dunlop et al. [7] the introduced switched beam directional antennas in WSNs, which reduces transmission delay, interference and flooding and therefore reduces the energy consumption. Gorman discussed the application of optimization algorithms in array beam pattern synthesis. They used Collaborative Beamforming (CB), a process by which multiple nodes collaborate with each other to form a distributed sensor array, transmit in the desired direction in such a way that constructive interference happens at the receiver. CB can result in energy savings for all

sensors involved in the network through the sharing of transmission overhead, thus improving the lifetime of the network. Hence CB requires directional antenna beam pattern using WSN array. A lot of emphases is placed on antenna array optimization. The beam characteristics, main lobe, SLL and beam-width are helpful in visualizing where antenna transmits and receives power. The beam pattern is produced by an array is dependent on the current feeding of each element. This synthesis technique applies various optimization algorithms which focuses on finding an excitation current vector which when applied to array elements, minimizes the beam ratio i.e. SLL to the main lobe.

The contents of the paper are divided into the following sections. Section 2 reviews directional beamforming for WSNs. In section 3, a novel beamformer for wideband beamforming with LMS algorithm is included. In section 4, simulations and results of wideband directional beamformer are discussed. Conclusions are drawn in section 5.

**Review of Directional Beamforming for WSN'S**

Kaiser, Dolph-Chebyshev two-parameter and Saramaki windows design for a given main lobe width and sidelobes [8]. The amplitude of the side lobes can be controlled using these windows, with respect to the main lobe. Fourier partial sums can be employed to approximate an arbitrary radiation pattern. Woodward-Lawson [9] sampling method is a well-known beam pattern synthesis method wherein sampling the desired pattern at pre-decided points is used to attain the radiation pattern. The conventional pattern synthesis methods are used to create a single or multiple main beam cosine patterns, which can be used for the perturbational method of Elliott [10] or the iterative method of Stutzman [11]. Half-power beam-width and the position of the main beam in the field characterizes the cosine-pattern function. The Woodward-Lawson methods and the Fourier series use the position of the main lobe and the half-power beam-width as the design parameters [12]. It was found that in the latter the SLL obtained was lower than the former method, but its SLL decays faster in Fourier series method. Dolph employed the Chebyshev polynomials to define the array weights. The developed closed-form solution resulted in a pattern which has all the sidelobes at equal levels. Tseng and Griffiths [13] discuss an iterative algorithm, which uses the solution of linearly constrained least-squares problems, for beam pattern synthesis. This algorithm can be used with any array geometry of real or complex weights and individual elements with differing responses. This method is robust and converges fast. The binomial array has very low SLL at the expense of directivity. To compromise between the uniform and binomial arrays Dolph used the first kind Chebyshev polynomial to design the pattern. First kind Chebyshev array has better directivity than the binomial array, but its SLL is lower than uniform array. Dolph-Chebyshev array has the equally distributed minor lobe levels, which reduces the beam efficiency. Phongcharoenpanich et al. [14] proposed an array pattern synthesis method using second kind Chebyshev polynomials and Legendre which show better beam efficiency and tapered minor lobe distribution. Taylor one-parameter [15] has better beam efficiency with tapered minor lobes, but the directivity is low and the method is not suitable for designing the array of odd number elements. Similarly second kind Chebyshev and Legendre show high beam efficiency and tapered minor lobes, but directivity reduces as compared to first kind of Chebyshev array. Though the first kind Chebyshev array doesn't possess tapered minor lobes, it is considered as the optimized array of beam-width and SLL. A method for pattern synthesis by achieving the tapered minor lobes for various applications like low noise and radar systems. Orthogonal polynomials like second kind of Chebyshev and Legendre are employed to design the antenna array pattern. One flaw of first kind Chebyshev array is that it yields uniform minor lobes which will allow signals to enter through those lobes [16]. Elliott [17] presented a method for pattern synthesis with uniform sidelobes. It is suitable only for uniformly distributed antenna arrays and cannot be directly applied for pattern synthesis of arrays with arbitrary element positions. El-Hajj et al. [18] presented a new method for calculation of directivity of Chebyshev arrays. This method is helpful in analyzing the directivity against varied SLL, inter-element phase shift and inter- element spacing for any given number of elements, but it doesn't provide any closed-form equation. The limitation is its dependence on excitation currents, which has to be evaluated every time the SLL and number of elements are changed. Dolph-Chebyshev array accomplishes pencil beam for a given number of antenna elements with a specified equiripple SLL. Freitas et al. [19] presented a method based on the modified Dolph-Chebyshev design which needs only a few parameters to be optimized. It is a simple method and used in wireless communications which require fast and adaptive algorithms for low sidelobe arrays. The Chebyshev approach provides an optimum solution when beam width is minimized for specified SLL [20]. Chebyshev array suffers from directivity saturation when the number of elements increases. Safaai-Jazi [21] introduced a better class of linear arrays which has equal SLL and can offer higher directivities than Chebyshev arrays with the same SLL and number of elements. Higher directivity can be achieved for a designed inter-element spacing and SLL when the number of elements is more than the threshold value. A non-recursive finite impulse response filter, namely Discrete Prolate Spheroidal (DPS) filter introduced by Tufts and Fraqcis [22] has superior performance characteristics when compared to filters in class. It has minimum ringing in time domain and maximum energy concentration is frequency passband. Many methods have been presented to design the weighting window which can create a flat top shaped beam in narrow-band conditions. The technique imposes a constraint on the magnitude response only in the main lobe region while controlling the sidelobes. The nonconvex pattern synthesis problem is implemented via iterative algorithms of convex optimization [23]. In recent years many papers have addressed the synthesis of antenna array pattern for wide-band conditions which has greater amount applications in medical field and under-water systems. Most of them rely on optimization of wideband band-pass by acting on the weighting window associated with the array. Curletto and Trucco [24] describe a technique to design a wideband beam pattern with the desired main lobe shape with acceptable SLL. Using simulated annealing (SA) algorithm they reproduce the main lobe profile as close as possible to a desired one. By comparing the spatial expression of radiation patterns of the

ULA and spectral expression of FIR filters, Zhang et al. [25] presented the relation between parameters in space domain of ULA and time domain for FIR. They achieved the flat-topped beam patterns with variable beam-width using Kaiser window, and the approach of FIR filter to ULA beamforming. Raza et al.[26 ] discuss the least square-based Eigen-filter method that has been implemented efficiently to both FIR filter and wideband beamformer. It involves calculating the resulting filter coefficients as the eigenvector of an appropriate Hermitian matrix and offers reduced complexity and less computing effort with greater numerical stabilization relative to the conventional least-squares method. The Eigen-filter technique is critically analyzed by exposing a serious performance problem in the passband of the designed FIR filter and the main lobe of the wideband beamformer, which occurs due to a formulation issue. A solution to mitigate this problem is then suggested by enforcing an extra constraint for controlling the response at the passband / main lobe, and design examples are given for both FIR filters and wideband beamformers to show the efficacy of the suggested technique.

In this paper, a weighting method employing suitable FIR filters is introduced for wideband beamformer suitable for WSN to be deployed linearly for achieving high directivity beams and to obtain wideband beamformer performance in the desired frequency band.

## Novel Weighting Method for WSN Wideband Beamformer
### Review of Conventional Wideband Beamformer

For the incoming wideband signal $x(n)$, the beamformer guides the main beam in the direction of arrival and produces nulls in the direction of interferers. Fig. 1, shows a wideband ULA beamformer with M antenna elements, inter-element spacing d of $0.5\lambda_{min}$ and filter length N for each element required to obtain frequency invariance over the desired frequency band.
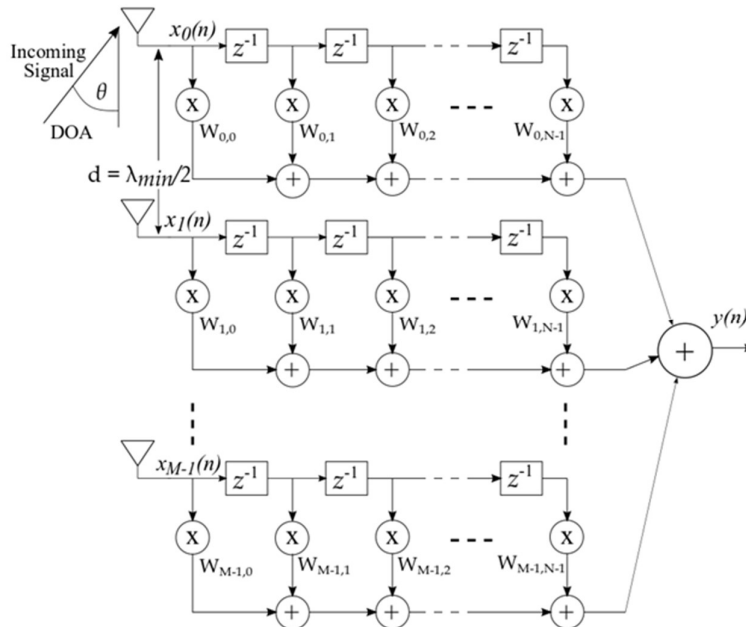


**Fig. 1: A M Element Wideband Beamformer.**

The current applied to a specific array element is $I_m$ and $\psi$ is the progressive phase difference. All the elements are assumed to be perfect isotropic radiators M and channel effects are ignored. The field pattern formed by an array for a direction of arrival angle $\theta$, the array factor is,

$$AF = \sum_{m=0}^{M-1} I_m e^{jm(2\pi cos\theta + \psi)}$$

(1)

The wideband beamformer frequency response as the function frequency $f$ and $\theta$ is,

$$H(f, \theta) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} W_{m,n} e^{-jm(2\pi f sin\theta/c)d} e^{-jn2\pi} \quad s$$

(2)

where $W_{m,n}$ is the $n^{th}$ coefficient or weight of the $m^{th}$ section, N taps in each section which are independent of M and $T_s$

is the sampling period. The total number of taps is equal to M × N. The filter coefficients $W_{m,n}$ are obtained by the 2-D Inverse Discrete Fourier Transform (IDFT) on $H(f, \theta)$ [27- 29].

## Obtaining Weights for Wideband Beamformer

In this section, a closed-form expression for impulse response coefficients of wideband beamformer or spatial filter which serves as the weighting function to shape the wideband beamformer response is proposed. The weights M and N for the wideband beamformer obtained from the filter design to shape the beamformer response and for frequency invariance for the given wideband of incoming signal frequencies. Also, the beamformer pattern should meet the specifications over the wide steering angle range.

The magnitude response function $H(f)$ of the beamformer is designed using trigonometric functions of frequency. The impulse response coefficients of the proposed filter $h(n)$ obtained are,

$$h(n) = \frac{sin\left(f_c(B_p + k)\right)}{2\pi(B_p + k)} + \frac{sin\left(f_c(B_p - k)\right)}{2\pi(B_p - k)} + \frac{\delta_p sin(f_z(B_t + k) - B_t\omega_c)}{2\pi(B_t + k)} + \frac{\delta_p sin(f_z(B_t - k) - B_t\omega_c)}{2\pi(B_t - k)}$$
$$- \frac{\delta_p sin(kf_c)}{2\pi(B_p + k)} - \frac{\delta_p sin(-kf_c)}{2\pi(B_p - k)} + \frac{\delta_s sin(\pi(B_s + k) - B_s f_s)}{4\pi(B_s + k)} + \frac{\delta_s sin(\pi(B_s - k) - B_s f_s)}{4\pi(B_s - k)}$$
$$- \frac{\delta_s sin(kf_s)}{4\pi(B_s + k)} - \frac{\delta_s sin(-kf_s)}{4\pi(B_s - k)}$$

(3)

where, $n = 0,1, \ldots M - \frac{1}{2}$, for M odd and $n = 0,1, \ldots \frac{M}{2} - 1$ for M even and $k = \frac{M-1}{2} - n$, $f_c$ is the passband, $f_z$ is the zero-crossing point of frequency response, $\delta_p$ is the passband ripple, and $\delta_s$ is the stopband level. $B_p$, $B_t$ and $B_s$ are filter design parameters which control the shape of magnitude function $h(f)$ in the passband, transition band and stop band respectively. The frequency invariance property of the beamformer over the desired frequency range is obtained by bandpass filter design [27] and weights M are designed. The proposed filter possesses low passband ripple and good stopband attenuation and a sharp transition to obtain a beamformer response with high directivity.

## Constrained LMS Optimization for Beamformer

The following section explains a novel wideband beamformer design with constrained Least Mean Squares (LMS) optimization. In the adaptive process, constrained LMS algorithm needed only the direction of arrival of the incoming signal and desired frequency response to progressively learn the statistics of noise arriving from directions other than look direction. The algorithm can retain a selected frequency response in the look direction while minimizing the output noise power.

The vector of tap voltages at the $k^{th}$ sample is specified as, $X(k)$ where $X^T(k) = [x_1(k), \ x_2(k), \ldots \ldots, x_{MN}(k)]$.
Sum of voltages due to the arriving signals in look direction and noises in the non-look direction makes the vector of tap voltages i.e. $X(k) = L(k) + N(k)$, where the vector of non-look-direction noises is $N^T(k) = [n_1(k), \ n_2(k), \ldots \ldots, n_{MN}(k)]$.
$W$ is the vector of weights at each tap, where $W = [w_1 \ w_2 \ \ldots \ w_{MN}]^T$.
At the time of the $k^{th}$ sample, the output of the array is $y(k) = W^T X(k) = X^T(k)W$.
The expected power output of the array is $E[y^2(k)] = W^T R_{XX} W$. The constraint matrix C can be written as $C = [c_1 \ldots c_N \ldots c_N]$. The constraint can now be written as $C^T W = F$, where Matrix F can be described as a vector of look-direction equivalent of a tapped delay line. The problem of system identification can, therefore, be described as follows,
$min \ \ W^T R_{XX} W$
$subject \ to \ C^T W$

(4)

By introducing a non-negative variable $\lambda$, the objective function can be converted to an equivalent optimization with existing constraints. $W_{opt}$ is obtained by the method of Lagrange multipliers.

$$H(w) = \frac{1}{2} W^T R_{XX} W + \lambda^T (C^T W - F)$$

(5)

where the Lagrange multiplier is $\lambda = -[C^T R_{XX}^{-1} C]^{-1} F$. Therefore the optimum weight vector can be written as $W_{opt} = R_{XX}^{-1} C [C^T R_{XX}^{-1} C]^{-1} F$ .

**Simulation and Results**

The WSN nodes with M omnidirectional antenna elements are deployed in a straight line with internode uniform spacing $d = 0.5\, \lambda_{min}$ to form a WSN ULA beamformer. Simulations are performed employing the proposed beamformer and the performance parameters are compared with existing beamforming techniques. The desired SLL is 40dB with a steering range of $-36^0$ to $36^0$ and the incoming wideband signal frequency of 1-3GHz . The proposed WSN ULA is simulated with M ranging from 9 to 15, to form highly directive beams to be received by other WSN nodes deployed in the network. Its performance for different antenna elements M and required SLL is compared for a given beam-width and SLL as shown in TABLE I. Fig 2 (a) shows the response of the beamformer with 15 elements and SLL of 27.5 dB, and steered by $36^0$ without any distortion in the main lobe. Fig 2 (b) shows the comparison of responses of proposed, Taylor and DPSS beamformers with M = 15. TABLE I shows that the proposed beamformer has better directivity and reduced SLL for a specified number of elements M compared to DPSS and Taylor beamformers. In proposed beamformer weighting shows tapering SLL characteristics hence narrow Transition Beam Width (TBW) and better First Null to Last Null ratio (FNLN) are obtained compared to other beamformers. Fig. 4 shows the wideband beam pattern of the proposed wideband beamformer to achieve the required beam pattern and steering capability for the incoming frequency 1-3GHz with M = 15 antenna elements. Also to obtain the frequency invariance performance in the frequency band of 1-3GHz, a bandpass filter [27] weighting is applied to the ULA beamformer with N=19 to obtain wideband beamformer as shown in Fig 4.

Constant gain is noted over the wideband 1-3GHz with SLL of 20dB and excellent steering capability of $-36^0$ to $36^0$ for the wideband beamformer. Directivity of the proposed beamformer with 15 elements measured in the desired direction of arrival, $0^0$ is 11.42. Fig 3 (a) shows proposed beamformer performance where directivity variation with the frequency and Fig 3 (b) shows higher directivity is achieved with increase in number of elements of the beamformer.

**Table 1: Comparison of Proposed Beamformer Performance with Existing Methods for the Number of Elements M = 9, 13, 15 and SLL 40dB**

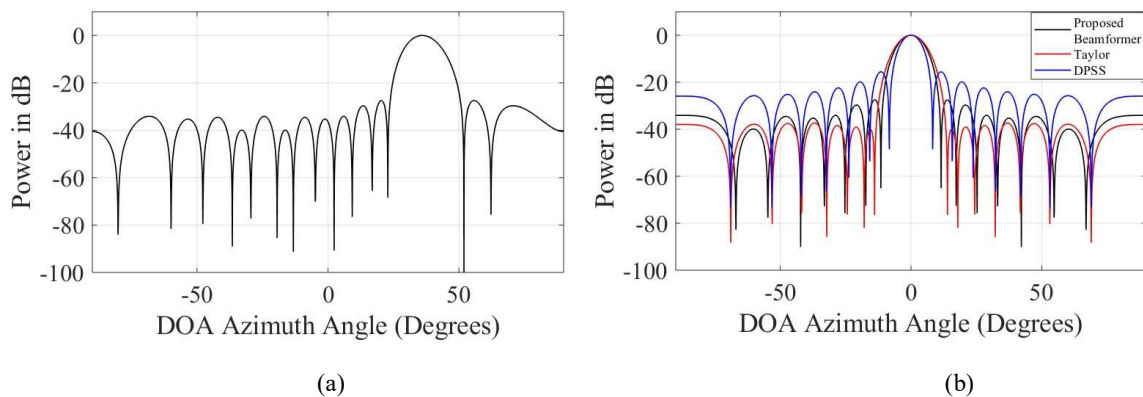| Filter Parameter | Proposed Filter | | | Taylor | | | DPSS | | |
|---|---|---|---|---|---|---|---|---|---|
| No. of Elements (M) | 9 | 13 | 15 | 9 | 13 | 15 | 9 | 13 | 15 |
| Transition Beamwidth (TBW) | $24.8^0$ | $21.2^0$ | $21.2^0$ | $45.1^0$ | $30.8^0$ | $26.7^0$ | $23.3^0$ | $16.1^0$ | $14.0^0$ |
| SLL(dB) | 16.8 | 22.7 | 27.5 | 40 | 40 | 40 | 15.6 | 15.5 | 15.55 |
| Half Power Beamwidth (HPBW) | $12.2^0$ | $9.4^0$ | $8^0$ | $15.8^0$ | $10.9^0$ | $9.4^0$ | $11.9^0$ | $8.4^0$ | $7.2^0$ |
| First Null to Last Null Ratio (FNLN) | 0.71 | 0.69 | 0.80 | 1.12 | 1.08 | 1.05 | 0.71 | 0.63 | 0.60 |
| Directivity | 9.81 | 11.09 | 11.42 | 8.83 | 10.43 | 11.05 | 9.86 | 11.46 | 12.09 |



(a)                                    (b)

**Fig. 2: (a) Response of Proposed Beamformer with the Number of Elements M = 15 with SLL 27.5dB and Steered by $36^0$.**
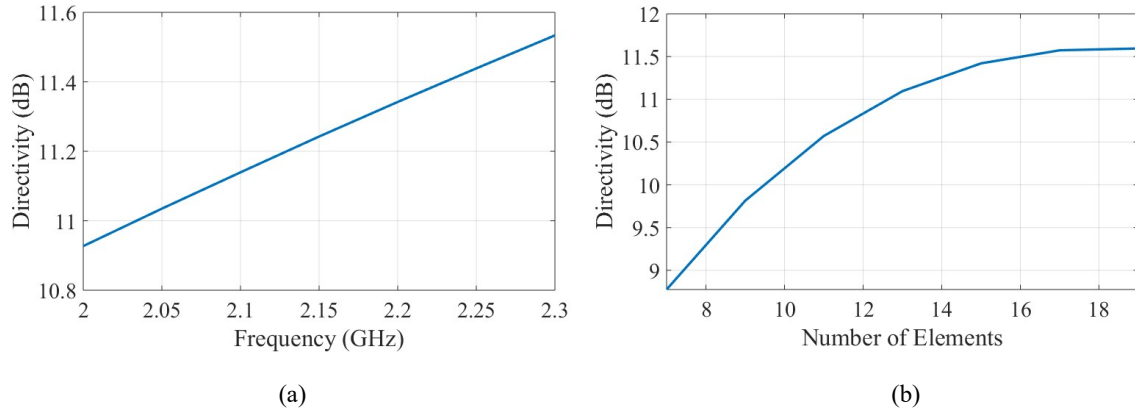**(b) Beampatterns of Proposed, Taylor and DPSS for M = 15 Elements.**

(a)

(b)

**Fig. 3: (a) Variation of Directivity of the Proposed Beamformer with Frequency.
(b) Variation of Directivity of the Proposed Beamformer with the Number of Elements.**
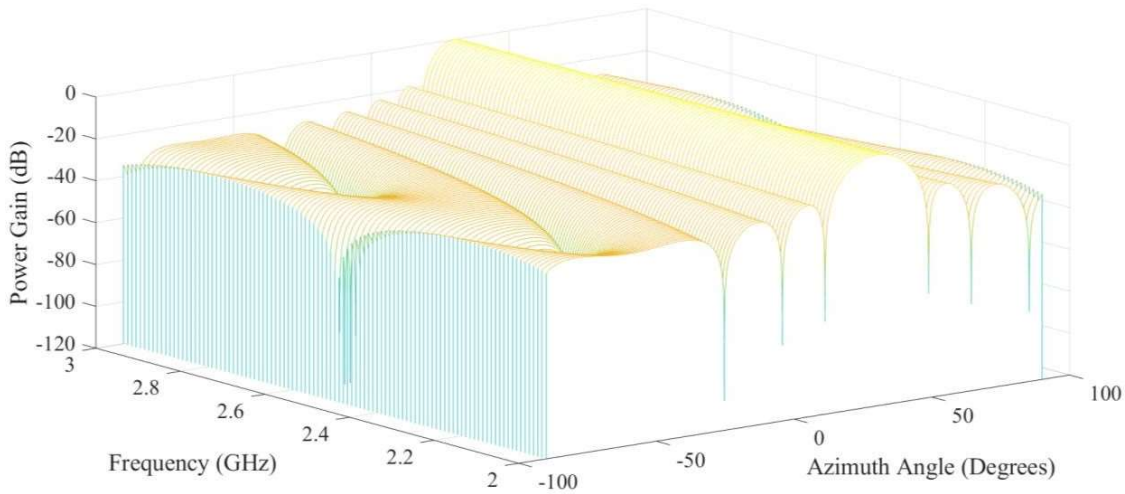


**Fig. 4: Performance of Proposed Wideband Beamformer with M = 15, SLL = 20dB and Steering Angle
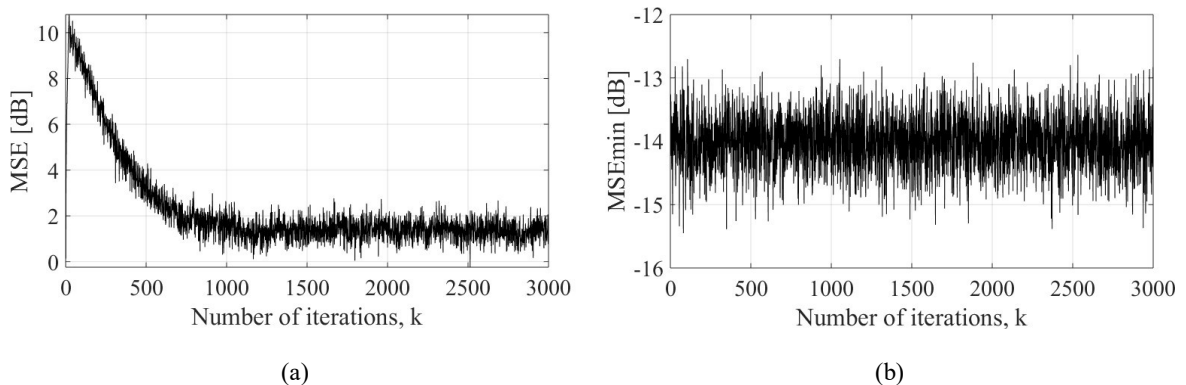= $30^0$**



(a)

(b)

**Fig. 5: (a) Learning Curve for MSE. (b) Learning Curve for MSE$_{min}$**

Fig 5 (a) and Fig 5 (b) show the progressive learning of the adaptive algorithm, which minimizes the error between actual and desired performance characteristics, as the number of iteration increases. It starts converging to the expected value by 20 iterations itself and reaches the result by 1000 iterations. From the figure, it is visible that the system identification algorithm minimizes the power density at no- look directions. Employing constrained LMS optimization techniques on the initial weights obtained from the proposed beamformer the SLL improves by 1dB. The wideband beamformer is

frequency invariant over the frequency of 1-3GHz by employing beamformer weights $W_m$ [27] where $m$ is the length of the wideband filter. It is found that beamformer exhibits frequency invariance property over the frequency band and steering angle of $-36^0$ to $36^0$.

**Conclusion**

In this paper, we propose a wideband beamformer for the implementation of ULA WSNs. A novel method is used to find the wideband beamformer weights and an optimized filter order is used to receive wideband signals at a wide range of steering angles and frequency invariance over wideband of 1-3GHz. As observed in simulation results good directivity is achieved in proposed weighting method of the beamformer. Constrained LMS optimization method is applied to the designed weights of the beamformer which further improves the SLL and it helps to achieve wide steering angles. An expression is derived for weights of the array factor of the beamformer, and SLL exhibits tapered minor lobes characteristics. This beamforming technique is suitable for wireless sensor network array since it exhibits good directivity property for point to point applications.

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