Review of Limit Equilibrium Methods for Stability Analysis of Dump Slope

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Abstract

OBD slope is artificial dump slope. OBD material is excavated material during extraction of the mineral or coal from the earth. Stability analysis of the overburden dump slope is crucial issue in mining industry. Assessment of the factor of safety of OBD slopes on the basis of the limit equilibrium methods are widely used in India. Limit equilibrium methods based on finding path of least resistance offered to sliding of the upper wedge of dump slope. Sliding surface may be circular or non-circular depends on the material of dump slope. Failures of overburden dump slope mostly will be circular. Circular failure occurs, when size of individual particles of fractured OBD will be very much less than overall size of slope. Discontinuities in the OBD slope are source of failure. Discontinuities are connected to each other and results into instability of the OBD slope. In this paper dealt with formulation of various limit equilibrium methods and its application to the calculation of factor of safety of the OBD slope. Overall comparison of limit equilibrium methods was done on the basis of free body diagram of failure surface wedge.

Introduction

Requirement of coal for power sector and other industries are increasing day by day. To fulfill this demand, mining activities go deeper and deeper. As depth of mining is increases then stripping ratio also increases rigorously. Huge amount of quantity of excavated material which is called as spoil is generated. This excavated material store in the form of waste dump or spoil dump also called as overburden dump. Dumps are again classified as internal dump (dump located inside the mine itself) and external dump (dump located nearby area of mine). Excavated waste material mainly consists of mixture of fragmented or fractured rock mass and soil. Main problem of dumping this waste material is stability. From past accidents, it is assume that stability of the dump become prime concerned. Dump should be design safely for long term stability with economical way. In Indian mining industries, external dumping preferred over internal dumping. By designing the dump slope in such a way that it should accommodate large capacity waste dump material. As slope angle of dump increased then dump waste volume carrying also increases but at the same time chances of failure of dump also increases. Design of overburden dump is depend on the geotechnical parameters of fractured rock mass and soil.[4] Overburden dumps are large man made slope constructed but with less attention to the analysis, design and implementation. As lack of attention for the designing of these dump results into the uneconomical and unsafe slope. Vibrations and blasting are disturbing forces for dump stability. Seismic forces cause the displacement of large mass of overburden and failure.

Observed Failure Mode and Failure Mechanism

Caldwell and Moss (1981) [3] mentioned various types of mine waste dump failure. Surface slide, shallow flow slide, base failure, block translation, Base failure and foundation circular failure. Analysis of overburden dump is complex in nature as its involve self weight of dump, heterogeneity of material (fractured rock to soil), geotechnical properties of dump material, strength of foundation over which dump crated, seepage pressure, tension cracks etc. Analytical techniques available are cumbersome to use. Mostly design of dump done on the basis of past experience. Rate of deformation and change of rate of deformation are good indicator for monitoring of the dump slope. Dump slope depend on the seepage i.e. pore pressure. Seepage is depends on the rainfall, porosity of the dump material and discontinuities. Location of ground water table i.e. phreatic line is most essential part of analysis of dump slope. Area of dump slope below the phreatic line is under pore pressure and due to this pore pressure effective strength of dump material reduces.[2] Again, water percolating through the overburden dump causes the erosion of dump material which results in to the dump failure. [12] Hence presence of ground water table in the overburden dump increases the driving force. Seismic or vibrations forces are induces the high inertial forces in the dump slope mass. Due do this vibrations forces the factor of safety reduces drastically and chances of failure of the dump slope increases. Earthquake load persist for very short period of time i.e. factor of safety reduces very short period of time. Overall effect of dump slope is accumulating the displacement caused by earthquake forces.

Factor affecting the Slope Stability [11]
1. Foundation soil – Nature of the foundation soil affect the stability of the slope. If the soft clay is present below the dump then chances of failure may increased.
2. Dump height – as dump height increase then factor of safety of the dump slope decreases.
3. Dump material – geotechnical properties of the dump material affect the porosity, cohesion, angle of internal friction, seepage condition etc.
4. Rainfall – this is most important factor that controlled the percolation of water from the dump. Seasonal variation in the ground water table will affect the stability of the slope.

Basically study of the failure mechanism of the fractured rock mass or soil is not done scientifically. Hence various combinations of configurations of the dump material are done which leads failure. Therefore following conditions are needs to studies
1. Conditions of occurrence of the types of failure mechanism
2. Conditions of failure initiation
3. Shape and location of the failure surface
4. Effect of dump slope geometry on failure mechanism

Objective of Slope Stability of Overburden Dump
1. To design the safe slope with given geotechnical properties of dump material and field constraints.
2. To find out the stability of existing dump from various methods of stability analysis.
3. To check the possibility of enhancement of capacity of existing dump.
4. To improved the stability of existing dump through reinforcement measures.

Design Parameters of Dump Slope [9]

Dump foundations properties	Geotechnical properties of dump material	Plasticity of dump material	Floor inclination
Bench width of dump	Bench height	Slope angle	Overall height of dump
Reduction of strength of dump material near the vicinity of the floor.		Strain development in dump	Construction sequence

Factors for failure of OBD Slope [9]

Gravity force of OBD material	Seepage forces	Degradations of the OBD slope material over a period of time	Vibrations (Earthquake, blasting, vehicular movement etc.)

Types of failure Mode [6][13]
1) Slope failure
    a) Face Failure
    b) Toe Failure
2) Base failure
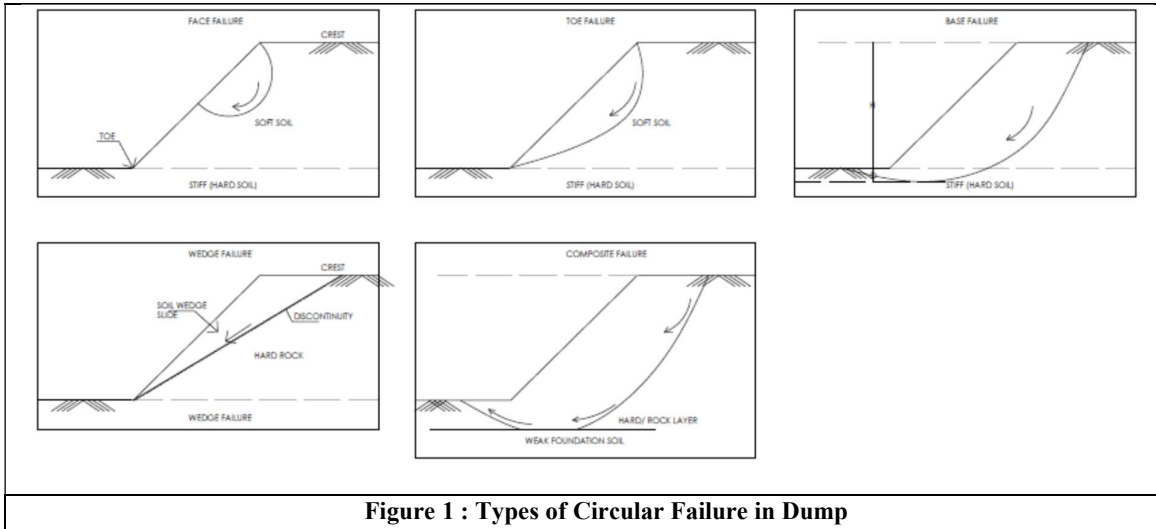3) Wedge failure
4) Composite failure

Figure 1 : Types of Circular Failure in Dump

Stability Analysis Procedure

Limit equilibrium method is used for the stability analysis of circular failure. In this procedure slope is divided into number of vertical slices and equilibrium analysis of individual slice is evaluated. The number equations of equilibrium available depend on number of slice (N) and number of equilibrium conditions that are used. The number equation available for force equilibrium is 2N. Number of equilibrium equations available if force and moment taken into consideration of analysis is 3N. Number of unknown if only force equilibrium consider is (3N-1) and (5N-1) if force and moment equations of equilibrium are considered. For realistic analysis, slope is divided into 10 to 40 numbers of slices. This analysis is statically indeterminate and hence, assumptions will require creating extra equations to evaluate all unknown parameters.

$$FS = \frac{Shear\ Strength\ available\ to\ resist\ sliding\ (c + \sigma \tan \phi)}{Shear\ stress\ required\ for\ equilibrium\ on\ the\ slip\ surface\ (\tau_f)}$$

Common Methods of Stability Analysis

The various methods are differ slightly from each other based on number of forces consider on slice but based on limit equilibrium method.

1) Stability of slope in c_u and $\phi_u = 0$
2) Swedish Circle method / Slip Circle method / Fellenius method / Slices method (1936)
3) Bishops's Simplified Method (1955)
4) Janbu's Method (1954)
5) Janbu's Corrected method (1973)
6) Morgenstern and Price's Method (1965)
7) Spencer's Method (1967)
8) Huang's Method (1980)
9) Stability Chart Method

Limit Equilibrium Analysis [14]

In this analysis we are equating the driving forces with resisting forces. Driving forces (Shearing forces) resulted from gravity forces and internal pressures acting on a mass bounded by failure surface. Gravity force is a function of weight of the block which likely to be failed, slope angle, depth of failure surface and some cases slope height. Pressure developed in the discontinuities of intact rock masses from water, freezing, swelling materials etc. and in case of fractured rock or soil, development of pressure due to presence of water in tension cracks. Resisting forces are provided by the shear strength along failure surface, and it is decreases as pore water pressure increases or by leaching of cementing constituents of slope material. Active and passive stresses are acting on a slope. The passive resistance at the lower portions is most significant.

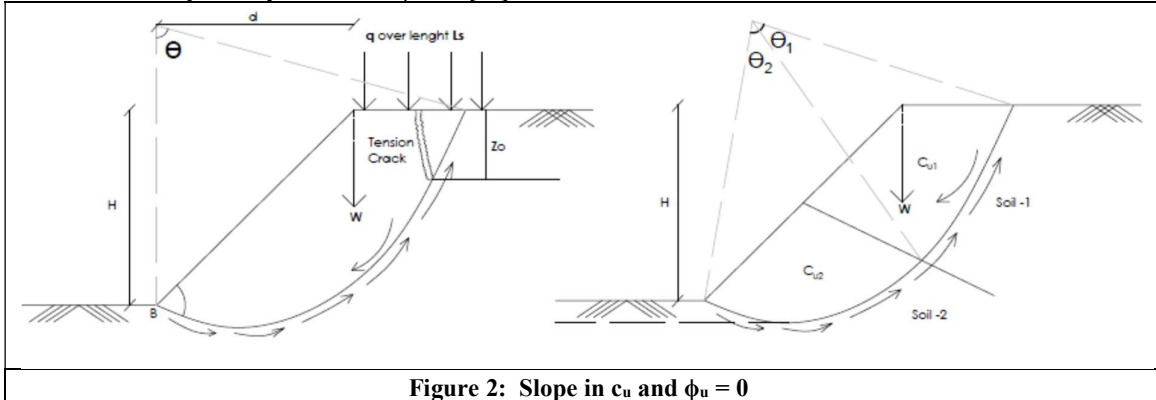**1.   Stability of Slope in c_u and $\phi_u = 0$ [17]**

<div align="center">

Figure 2: Slope in c_u and $\phi_u = 0$

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$$Wxd - \int_o^\theta dl \; x \; c_u \; x \; R = 0$$

$$Wxd - \frac{c_u R}{F} \int_o^\theta dl \; = 0$$

$$Wxd - \frac{c_u R^2 \theta}{F} = 0$$

$$F = \frac{c_u R^2 \theta}{Wd}$$

$$F = \frac{c_u L_a R}{Wd}$$

q is a surcharge loading applied over the length of Ls
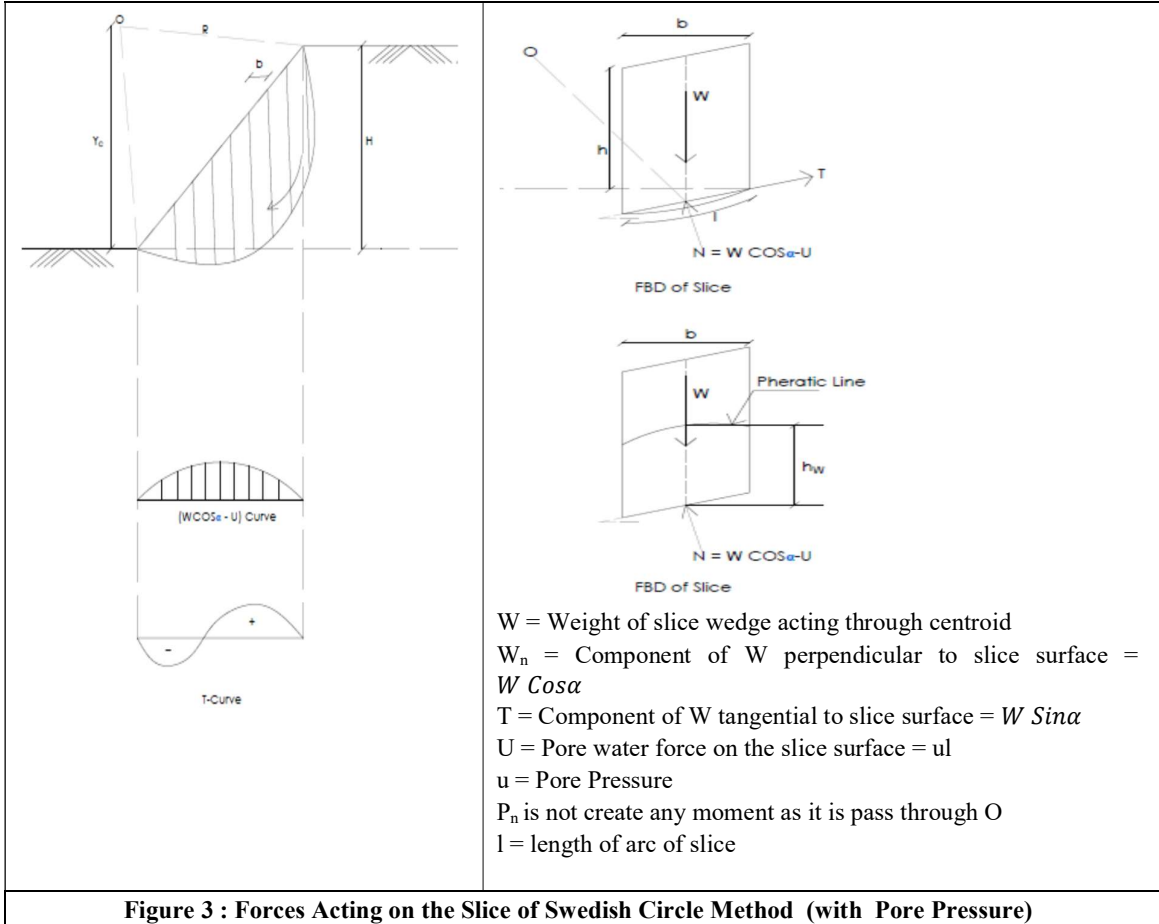
$$F = \frac{c_u R^2 \theta}{Wd + qL_s}$$

If soil is composed of two different layers as indicated in the Fig 2, then

$$F = \frac{R^2 (c_{u1}\theta_1' + \; c_{u2}\theta_2')}{Wd + qL_s}$$

2. Swedish Circle Method / Slip Circle Method / Fellenius Method / Slices Method [5][7]
Fellenius published a method of slices based on cylindrical failure Surface which Slip Circle method / Fellenius method. This method is modified for effective stresses (considering pore pressure) called as Ordinary method of slices. Assumptions
   1.   Slip surface assumed as an arc of a circle.
   2.   Method based on total stress analysis can be extended for effective stress analysis.
   3.   It is for homogeneous, isotropic mass without seepage.
   4.   Minimum factor of safety decided by trials.
   5.   Wedge divided into number of slices.
   6.   Interslice forces may or may not be accounted.
   7.   Both total stress as well as effective stress analysis can be done.
   8.   Method can be applicable to
        a.   Homogeneous soil
        b.   Stratified soil
        c.   Saturated, semi-saturated, submerge soil masses with or without seepage
        d.   Uniform / Non uniform slopes

W = Weight of slice wedge acting through centroid

W_n = Component of W perpendicular to slice surface = $W\,Cos\alpha$

T = Component of W tangential to slice surface = $W\,Sin\alpha$

U = Pore water force on the slice surface = ul

u = Pore Pressure

P_n is not create any moment as it is pass through O

l = length of arc of slice

Figure 3 : Forces Acting on the Slice of Swedish Circle Method (with Pore Pressure)

M_D = Driving Moment @ O = $r \Sigma T$

M_R = Resisting Moment @ O = $\Sigma r[c'l + (Wn - U)tan\phi']$

$$fos = \frac{MR}{MD}$$

$$fos = \frac{\Sigma[c'l+(Wn-\ \ )tan\phi']}{\Sigma T}$$

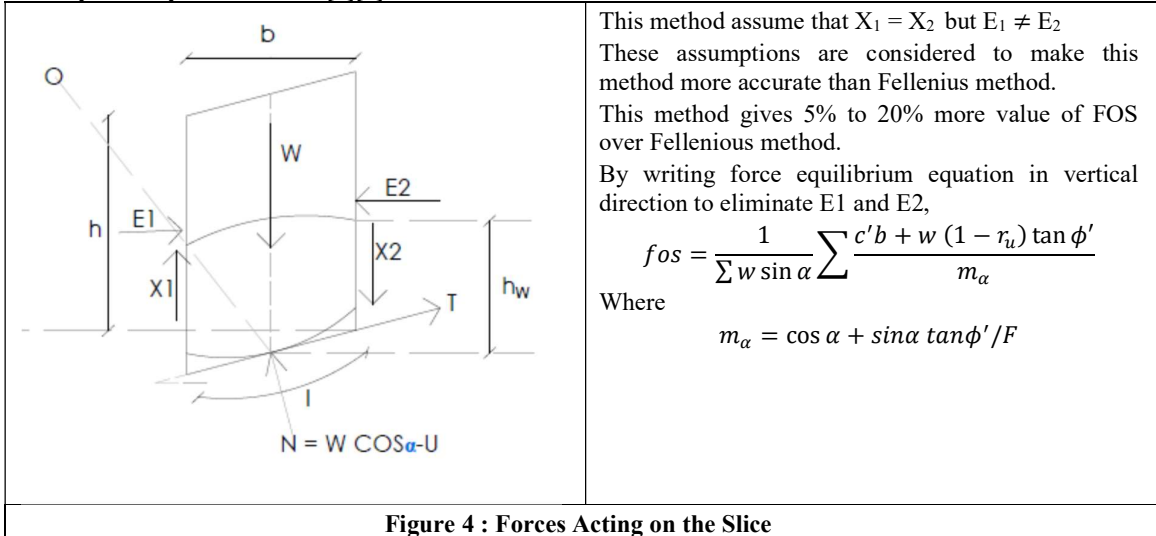$$fos = \frac{\Sigma[c'l+(W\cos\alpha\ \ \ \ )tan\phi']}{\Sigma W\sin\alpha}$$

For dimensionless term $r_u = \frac{ub}{w} = \frac{ub}{\gamma bh}$

$$= \frac{\gamma_w\,h_w}{\gamma h}$$

h_w is obtained at mid height of slice with flow net.

$$fos = \frac{\Sigma[c'l + (w\cos\alpha - ul)\ tan\phi']}{\Sigma W\sin\alpha}$$

$$fos = \frac{\Sigma[c'l + w\,(\cos\alpha - r_u\sec\alpha)\,tan\phi']}{\Sigma W\sin\alpha}$$

Bishops's Simplified Method[1][2]

This method assume that $X_1 = X_2$ but $E_1 \neq E_2$

These assumptions are considered to make this method more accurate than Fellenius method.

This method gives 5% to 20% more value of FOS over Fellenious method.

By writing force equilibrium equation in vertical direction to eliminate E1 and E2,

$$fos = \frac{1}{\sum w \sin \alpha} \sum \frac{c'b + w\,(1 - r_u)\tan \phi'}{m_\alpha}$$

Where

$$m_\alpha = \cos \alpha + \sin\alpha\,\tan\phi'/F$$

Figure 4 : Forces Acting on the Slice

In this method, initial value of FOS is assumed (slightly greater than FOS of Fellenius method) and substitute in the equation to obtained new FOS. This process of iteration is repeated until assumed FOS and calculated FOS nearly equal. Only 3-4 trial are required to get the appropriate lowest value of FOS and process is rapid.

Bishop's also show that E1-E2 ≠ 0 of resultant has only marginal effect on FOS.

Janbu's Method [8]

In this method applicable to any shape of failure surface. This analysis is based on slice method. All equilibrium conditions are fully satisfied. This method is similar to Bishop's simplified method, a minimum FOS is obtained for critical non-circular failure surface. This method required more computational efforts

Janbu's General Equation for FOS is as below

.

$$fos = \frac{1}{\sum w \tan \alpha} \sum \frac{c'b +\ (w - ub + \Delta X)\tan \phi'}{m_{\alpha_j}}$$

Where

$$m_{\alpha_j} = cos^2\alpha\,[1 + \frac{\tan \alpha\ tan\phi'}{F}]$$

A method is suitable for routine applications which based on neglecting ΔX term and increasing the number of slices. To simplify the calculations, the sliding mass divided into blocks and then each block is divided into number of slices. The angle α is same for all slices but different pore pressure in particular block. Application of routine method will cause the reduction in factor of safety corresponding to the critical surface. Therefore Janbu's proposed increased in factor of safety as per the for c' = 0, c'-ϕ type of soil and ϕ = 0 which is based on the ratio of d/L.

L =length of AE

d = maximum distance of non- circular failure surface from line AE.

Increased factor of safety given as

$$fos_{corrected} = f_0 . fos_{before\ correction}$$

$$f_0 = 1 + K[\frac{d}{L} - 1.4\frac{d^2}{L}]$$

For

c' = 0	Then	K = 0.31
c' > 0 and ϕ' > 0	Then	K = 0.5

Janbu's Corrected Method [8]
1. This method also called as Junbu's modified method of slice / Janbu's generalized method of slice (GPS).
2. It handles complicated geometry and failure surface.
3. Consider interslice forces and assume line of action thrust for interslice forces.

4. It's satisfied both force and moment equations.
5. This method satisfied all conditions of equilibrium of moment for slice – vertical force and horizontal force, moment equilibrium of whole sliding mass except the last slide.
6. Interslice forces may be calculated by considering moment about centre of base of slice.
7. Thrust line it is line of location of interslice forces acts therefore problem become statically determinate.
8. PGS, line of thrust defined as, the line joining the points of application of resultant of interslice forces in respect of all slides.
9. As per Janbu's (1957) and Spencer (1973), line of thrust lies between 1/3rd to 2/3rd of the slice height from the base in active and passive condition respective (Aryal, 2008).

$$fos_m = \frac{\sum\{c'l + (N - ul)tan\phi'\}}{\sum w \, sin\alpha}$$

$$fos_f = \frac{\sum\{c'l + (N - ul)tan\phi'\} \, sec\alpha}{\sum\{w - (X2 - X1)\} \, tan\alpha + \sum(E2 - E1)}$$

Total base Normal force acting on shear surface is a function of interslice shear force.

$$N = \frac{\{w - (X2 - X1) - \frac{(c'l - ul \, tan\phi')sin\alpha}{F}\}}{m_\alpha}$$

Morgenstern and Price's Method [10]
1. Sliding mass divided into the vertical slices.
2. Failure surface is non-circular

Equilibrium of vertical element of width dx within the each slice is then investigated by introducing five functions as follows
1. Y= s(x) defines shape of the slope and its boundary (its linear equation)
2. Y = y(x) represents failure surface (for each slice linear equation is assumed)
3. Y = h(x) defines the line of action of forces due to water pressure
4. Y = y'(x) defines of line of action of effective normal force E' acting on vertical side.
5. X= λf(x) E' defines the line of action of shear stress to X and effective normal force E'

 Force equilibrium equations constructed parallel and normal to the direction to the sliding surface, are combined with an application of Mohr-Coulmb failure criteria on the failure plane to yield following equations.
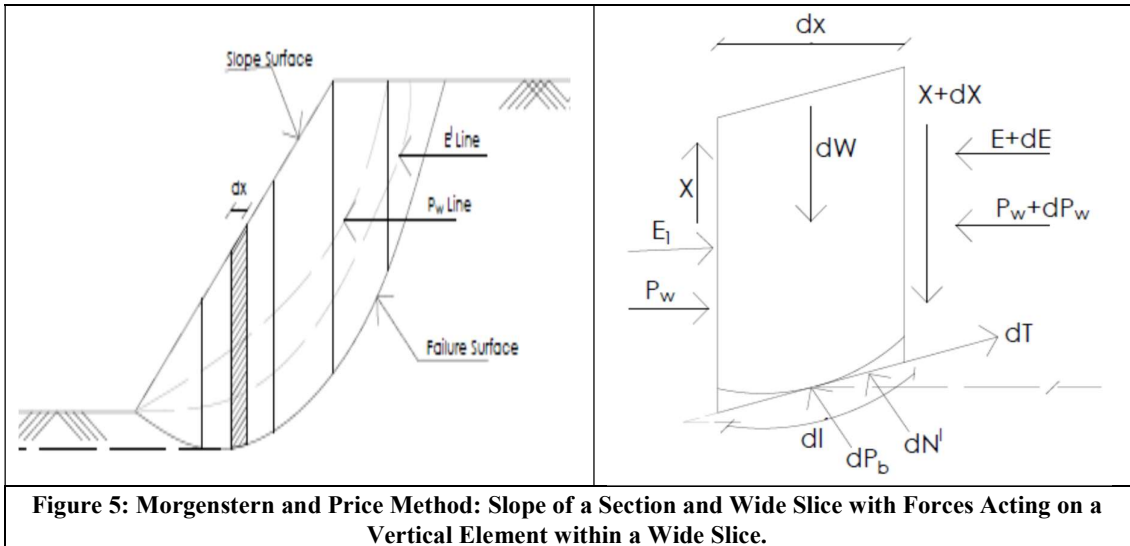

Figure 5: Morgenstern and Price Method: Slope of a Section and Wide Slice with Forces Acting on a Vertical Element within a Wide Slice.

$$\frac{c'}{F} \sec^2 \alpha - \frac{\tan \phi'}{F} \left[\tan \alpha \frac{dE'}{dx} + \tan \alpha \frac{dP_W}{dx} + \frac{dX}{dx} - \frac{dW}{dx} + \sec \alpha \frac{dP_b}{dx} \right.$$
$$= \frac{dE'}{dx} + \frac{dP_W}{dx} - \tan \alpha \frac{dX}{dx} + \tan \alpha \frac{dW}{dx}$$

For convenience moment equilibrium is considered at the midpoint of the base of the element and yields the following differential equations

$$y \left(\frac{dE'}{dx} + \frac{dP_w}{dx} \right) - \left[\frac{d(E'^{y'}_t)}{dx} + \frac{d(P_w h)}{dx} \right] + X = 0$$

The difficulties in the solving these two differential equations may be overcome by considering the linear equations for all five elements within the integration limit of slice.

In above equations

λ is scale factor and determined within the solution.

Pw is the water pressure acting on the vertical plane of each slice.

Spencer's Method [15][16]
1. This analysis is based on slice method.
2. This analysis is applicable to failure surface of any shape.
3. This analysis satisfied all conditions of equilibrium

The interslice forces E1, E2, X1 and X2 are replaced by single force Z with its point of application on the centre line of the slice and making angle Θ with horizontal. In general formulation it's assume that direction of Z is opposite to the direction of T. Vertical and horizontal force equilibrium equations are combined to eliminate N' and gives following equation

$$Z = \frac{\left(\frac{c'l}{F} \right) + \frac{(W \cos\alpha - ul)\tan\phi'}{F} - W\sin\alpha}{\cos(\alpha - \theta)\left[1 + \frac{\tan(\alpha - \theta)\tan\theta'}{F} \right]}$$

.Again

$$\Delta X = Z \sin \theta$$
$$\Delta E = Z \cos \theta$$

And

$$\sum Z \sin \theta = 0 \qquad \sum Z \cos \theta = 0$$

Moment of rotation about centre of rotation is zero

$$\sum Z \cos (\alpha - \theta) = 0$$

Results and Discussion
1. Bishops's Simplified Method gives 5% to 20% more value of FOS over Fellenius method. In Bishops's Simplified Method is assumed equilibrium of vertical forces between slices.
2. Janbu's Method In this method applicable to any shape of failure surface. This method is similar to Bishop's simplified method, a minimum FOS is obtained for critical non-circular failure surface.
3. Janbu's Corrected methods consider interslice forces and assume line of action thrust for interslice forces. This method satisfied all conditions of equilibrium of moment for slice – vertical force and horizontal force, moment equilibrium of whole sliding mass except the last slide.
4. Morgenstern and Price's Method Sliding mass divided into the vertical slices. Failure surface is non-circular.
5. Spencer's Method analysis is based on slice method. This analysis is applicable to failure surface of any shape. This analysis satisfied all conditions of equilibrium

Conclusion
In this procedure slope is divided into number of vertical slices and equilibrium analysis of individual slice is evaluated. In this analysis we are equating the driving forces with resisting forces. Driving forces (Shearing forces) resulted from gravity forces and internal pressures acting on a mass bounded by failure surface. Gravity force is a function of weight of the block which likely to be failed, slope angle, depth of failure surface and some cases slope height. Pressure developed in the discontinuities of intact rock masses from water, freezing, swelling materials etc. and in case of fractured rock or soil, development of pressure due to presence of water in tension cracks. Resisting forces are provided by the shear strength along failure surface, and it is decreases as

pore water pressure increases or by leaching of cementing constituents of slope material. Active and passive stresses are acting on a slope.

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